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Technical Note

On the role of the fluid effusivity in unsteady heat transfer between a boundary layer and a solid wall

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ABSTRACT

The problem of obtaining the wall heat flux in the presence of unsteady heat transfer in two-dimensional, turbulent boundary layer flow is re-examined. A novel expression to produce estimates of the amplitude of the fluctuating wall heat flux has been proposed for the foregoing conditions. This expression is based on flow field measurements instead of measurements in the solid wall substrate, thus allowing us to take the flow dynamics directly into account in the analysis. The fluid effusivity, a measure of the ability of the fluid to exchange thermal energy with its surroundings, was shown to be the dominant parameter controlling the unsteady heat transfer process.

1. Introduction

The study of unsteady heat transfer with wall surfaces under forced convection constitutes a topic of great interest since the walls of many thermal systems are often subjected to velocity and temperature fluctuations in the flow, as well as to time variations of their thermal boundary conditions. The experimental estimation of the unsteady heat flux $\Phi_w(t)$ exchanged between the fluid flow and a wall surface is known to involve the effusivity of the solid wall substrate. Fluid flow characteristics such as the velocity fluctuations and the fluid effusivity are not directly taken into account as a general rule. Analysing the unsteady heat transfer problem from the fluid side is complicated by the presence of time-dependent velocity and temperature gradients in the flow, which are non-linearly related. Nevertheless, it is obvious that the effusivities of both the fluid and wall material have an important role on the analysis of the underlying fluctuating heat transfer mechanisms. On the other hand, by carrying out the analysis solely on the wall side, the fluid flow phenomena are disregarded. As a consequence, erroneous results are likely to be obtained unless complex corrections are made due to thermal inertia of the material, deficient surface contact and measurement uncertainties [1].

It has been shown that the importance of the effusivity cannot be circumvented in the study of the heat transfer due to brief thermal contact between different bodies [2]. In the presence of fluid flow, we may choose to work with the effusivity of the fluid e, given by

$$e = \sqrt{\rho c_p k} \tag{1}$$

where ρ, c_p and k denote the fluid density, specific heat capacity and thermal conductivity, respectively. The effusivity of a material is a measure of its ability to exchange thermal energy with its surroundings, thus affecting the contact temperature between two bodies at different temperatures [3]. It is governing the resulting conductive heat transfer inside the wall substrate [4]. The fundamentals on this matter are widely described in the literature but various extensions to the theory have also been made in past years [5–8]. These studies suggest that focusing the analysis on the fluid rather than on the solid may be useful to improve the quality of unsteady heat transfer estimates between the fluid flow and a wall surface, taking into account the instantaneous velocity and temperature characteristics in the flow. Naturally, heat transfer calculations in the flow can be attempted using correlations based on convective heat transfer coefficients [9], though transient effects are not accounted for. This is made at the expense of a real instantaneous approach, sacrificing details on the flow velocity and temperature fields. As a consequence, such approximate methodology fails to predict important phenomena in the presence of unsteady flow fluctuations because a quasi-steady state is implicitly assumed. For example, this procedure has shown to be unable to anticipate the occurrence of thermal stripping in tee-junctions [10–11]. Hence, this paper aims at improving our ability to quantify the unsteady heat transfer mechanisms at a wall surface subjected

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Nomenclature

c_p	specific heat capacity of the fluid
e	effusivity of the fluid, $\sqrt{\rho c_p k}$
k	thermal conductivity of the fluid
RMS	root mean square
t	time
T	temperature
U	instantaneous fluid velocity in x-direction
V	instantaneous fluid velocity in y-direction
x	longitudinal coordinate
y	wall-normal coordinate
Greek symbols	
α	thermal diffusivity of the fluid, $k/\rho c_p$
δy	small wall distance
Φ	heat flux
ρ	density of the fluid
Subscripts	
δy	at a location $y = \delta y$
w	at the wall surface ($y = 0$)
Superscripts	
'	temporal fluctuation
–	time-average

to forced convection by proposing an expression for the wall heat flux fluctuation amplitude Φ'_w, which depends exclusively on flow field quantities and fluid variables. A brief discussion on the physical significance of the different terms appearing in this expression is also presented.

2. Heat flux formulations

The unsteady heat transfer between a two-dimensional, unsteady, turbulent boundary layer flow and a flat wall is the case under study. It is schematically illustrated in Fig. 1, showing that the main flow occurs along the x-direction, i.e., along the wall. The physical properties of the fluid are assumed to be constant and the flow is treated as incompressible. In addition, heat losses by viscous dissipation are neglected. No slip conditions at the wall are considered, hence the fluid velocity components $U = V = 0$ at the wall surface. The heat flux between the flow and the wall is assumed to propagate in the y-direction, i.e., perpendicularly to the wall. The wall surface is located at $y = 0$, whereas $y > 0$ is the flow region and $y < 0$ is the area of the solid wall substrate.

An expression is sought for the amplitude of the fluctuating heat flux at the wall Φ'_w. It is usually represented by the root mean square (RMS) of the unsteady heat flux at the wall $\Phi_w(t)$ and written as $\Phi'_w = \mathrm{RMS}(\Phi_w(t))$, where t is time. At the wall, $\Phi_w(t)$ is a conductive flux, which is proportional to the thermal conductivity and the instantaneous temperature gradient in the wall-normal direction. Denoting the unsteady conductive flux at a generic wall distance y by $\Phi_y(t)$, the instantaneous fluid temperature by $T(t)$, its temporal fluctuation by $T'(t)$, and using the decompositions $T(t) = \overline{T} + T'(t)$ and $\Phi_w(t) = \overline{\Phi_w} + \Phi'_w(t)$, where an overbar indicates a time average, we may write

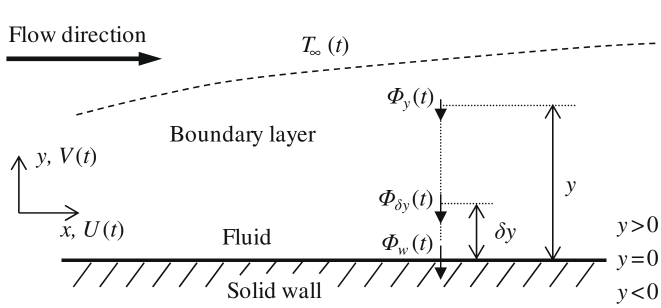

Fig. 1. Schematics of the unsteady, turbulent heat transfer problem under study.

$$\Phi_y(t) = -k\left(\frac{\partial T(t)}{\partial y}\right)_y, \quad \Phi_w(t) = -k\left(\frac{\partial T(t)}{\partial y}\right)_{y=0} \tag{2}$$

$$\left(\Phi'_w\right)^2 = \left(\mathrm{RMS}(\Phi_w(t))\right)^2 = \overline{\left(-k\left(\frac{\partial T'(t)}{\partial y}\right)_{y=0}\right)^2} \tag{3}$$

The wall heat flux is defined at the location $y = 0$ but, for the purpose of an experimental determination, it is extrapolated by means of a truncated Taylor series near the wall, i.e., at a location $y = \delta y$ in the fluid ($\delta y > 0$). The accuracy of this approximation depends only on the remainder terms of the series development. Therefore, if δy is small enough, the error is minimised.

The unsteady conductive heat flux at the location $y = \delta y$ will henceforth be denoted by $\Phi_{\delta y}(t)$ using the definition given in Eq. (2). So, it can be shown (see Appendix A) that an approximated expression for $\left(\Phi'_w\right)^2$ involves a simple combination of terms in $\left(\Phi'_{\delta y}\right)^2 \equiv \overline{\left(\Phi'_{\delta y}(t)\right)^2}$, i.e.,

$$\left(\Phi'_w\right)^2 \approx \left(\Phi'_{\delta y}\right)^2 - \frac{\partial}{\partial y}\left(\Phi'_{\delta y}\right)^2 \delta y \tag{4}$$

This expression is essentially an Euler approximation. Its range of validity is, again, determined by the remainder term of the Taylor series. As required for consistency, in the limit when $\delta y \to 0$, the fluctuating amplitudes Φ'_w and $\Phi'_{\delta y}$ exhibit the same behaviour as long as the first derivative appearing in Eq. (4) is continuous, which in fact constitutes a physical requirement.

3. Estimation of the fluctuating heat flux amplitude

In this section, the energy balance equation (without heat sources) at the location $y = \delta y$ is used to derive an expression for the estimation of the amplitude of the fluctuating heat flux at the wall. As before, a decomposition of the form $A(t) = \overline{A} + A'(t)$ is applied to all field variables, leading to

$$\rho c_p\left(\frac{\partial \overline{T} + T'(t)}{\partial t} + \left(\overline{U} + U'(t)\right)\frac{\partial \overline{T} + T'(t)}{\partial x} + \left(\overline{V} + V'(t)\right)\frac{\partial \overline{T} + T'(t)}{\partial y}\right)_{\delta y} = k\left(\frac{\partial^2 \overline{T} + T'(t)}{\partial x^2} + \frac{\partial^2 \overline{T} + T'(t)}{\partial y^2}\right)_{\delta y} \tag{5}$$

For the sake of brevity, the spanwise fluctuations have been omitted from the analysis because their presence will not change the final result, as the boundary layer simplifications later applied to terms involving velocity fluctuations in the main flow direction also apply to their counterparts involving the spanwise fluctuations. In addition, for the two-dimensional boundary layer problem introduced in the previous section, the mean spanwise velocity is zero. By subtracting from Eq. (5) its time-averaged counterpart, we obtain the following equation for the fluctuating temperature field:

$$\rho c_p\left(\frac{\partial T'(t)}{\partial t}+\overline{U}\frac{\partial T'(t)}{\partial x}+U'(t)\frac{\partial \overline{T}}{\partial x}+U'(t)\frac{\partial T'(t)}{\partial x}-\overline{U'(t)\frac{\partial T'(t)}{\partial x}}+\overline{V}\frac{\partial T'(t)}{\partial y}+V'(t)\frac{\partial \overline{T}}{\partial y}+V'(t)\frac{\partial T'(t)}{\partial y}-\overline{V'(t)\frac{\partial T'(t)}{\partial y}}\right)_{\delta y}=k\left(\frac{\partial^2 T'(t)}{\partial y^2}+\frac{\partial^2 T'(t)}{\partial x^2}\right)_{\delta y} \tag{6}$$

Aiming to get to a simplified expression, it is assumed that the classical (turbulent) boundary layer relations for an order of magnitude analysis are applicable from the wall to $y = \delta y$, namely

$$\overline{V} \ll \overline{U}, \quad U'(t) \approx V'(t) \tag{7a}$$

$$\frac{\partial \overline{T}}{\partial x} \ll \frac{\partial \overline{T}}{\partial y}, \quad \frac{\partial^2 \overline{T}}{\partial x^2} \ll \frac{\partial^2 \overline{T}}{\partial y^2}, \quad \frac{\partial T'(t)}{\partial x} \ll \frac{\partial T'(t)}{\partial y}, \quad \frac{\partial^2 T'(t)}{\partial x^2} \ll \frac{\partial^2 T'(t)}{\partial y^2} \tag{7b}$$

By combining the order of magnitude relations given in Eqs. (7a) and (7b), it is possible to obtain new relations corresponding to the terms appearing in Eq. (5) that are involved in convective heat transfer, i.e.

$$U'(t)\frac{\partial \overline{T}}{\partial x} \ll V'(t)\frac{\partial \overline{T}}{\partial y}, \quad U'(t)\frac{\partial T'(t)}{\partial x} \ll V'(t)\frac{\partial T'(t)}{\partial y}, \quad \overline{U'(t)\frac{\partial T'(t)}{\partial x}} \ll \overline{V'(t)\frac{\partial T'(t)}{\partial y}} \tag{8}$$

Based on the foregoing analysis, negligible terms are subsequently dropped in Eq. (6), yielding the following approximated version of the energy balance equation at $y = \delta y$:

$$\rho c_p\left(\frac{\partial T'(t)}{\partial t}+\overline{U}\frac{\partial T'(t)}{\partial x}+V'(t)\frac{\partial \overline{T}}{\partial y}+V'(t)\frac{\partial T'(t)}{\partial y}-\overline{V'(t)\frac{\partial T'(t)}{\partial y}}\right)_{\delta y} \approx k\left(\frac{\partial^2 T'(t)}{\partial y^2}\right)_{\delta y} \tag{9}$$

Multiplying Eq. (9) by the temporal fluctuating temperature $T'(t)$ and using the following mathematical property:

$$\left(\frac{\partial^2 T'}{\partial y^2}\right)T' = \left(\frac{1}{2}\frac{\partial^2 (T')^2}{\partial y^2}-\left(\frac{\partial T'}{\partial y}\right)^2\right) \tag{10}$$

we obtain the alternative formulation

$$\rho c_p\left(\frac{1}{2}\frac{\partial (T'(t))^2}{\partial t}+\frac{\overline{U}}{2}\frac{\partial (T'(t))^2}{\partial x}+V'(t)T'(t)\frac{\partial \overline{T}}{\partial y}+\frac{V'(t)}{2}\frac{\partial (T'(t))^2}{\partial y}-T'(t)\overline{V'(t)\frac{\partial T'(t)}{\partial y}}\right)_{\delta y} \approx k\left(\frac{1}{2}\frac{\partial^2 (T'(t))^2}{\partial y^2}-\left(\frac{\partial T'(t)}{\partial y}\right)^2\right)_{\delta y} \tag{11}$$

The time average of Eq. (11) yields

$$\rho c_p\left(\frac{\overline{U}}{2}\frac{\overline{\partial (T'(t))^2}}{\partial x}+\overline{(V'(t)T'(t))}\frac{\partial \overline{T}}{\partial y}+\frac{\overline{V'(t)}}{2}\frac{\overline{\partial (T'(t))^2}}{\partial y}\right)_{\delta y} \approx k\left(\frac{1}{2}\frac{\overline{\partial^2 (T'(t))^2}}{\partial y^2}-\overline{\left(\frac{\partial T'(t)}{\partial y}\right)^2}\right)_{\delta y} \tag{12}$$

which can be rewritten in a more compact form, as follows:

$$\rho c_p\left(\frac{\overline{U}}{2}\frac{\overline{\partial (T'(t))^2}}{\partial x}+\overline{(V'(t)T'(t))\frac{\partial T(t)}{\partial y}}\right)_{\delta y} \approx k\left(\frac{1}{2}\frac{\overline{\partial^2 (T'(t))^2}}{\partial y^2}-\overline{\left(\frac{\partial T'(t)}{\partial y}\right)^2}\right)_{\delta y} \tag{13}$$

Finally, estimates for the amplitude of the fluctuating wall heat flux Φ'_w may now be obtained from Eq. (4), noting that the following expression for $\Phi'_{\delta y}$ can be found using Eq. (13):

$$\Phi'_{\delta y} \approx e\sqrt{\left(\alpha\left(\frac{1}{2}\frac{\overline{\partial^2 (T'(t))^2}}{\partial y^2}\right)_{\delta y}-\left(\frac{\overline{U}}{2}\frac{\overline{\partial (T'(t))^2}}{\partial x}+\overline{(V'(t)T'(t))\frac{\partial T(t)}{\partial y}}\right)\right)_{\delta y}} \tag{14}$$

where α stands for the thermal diffusivity and the effusivity of the fluid e, previously defined in Eq. (1), appears explicitly. However, it must be pointed out that the application of Eq. (4) for the stated purpose also involves the estimation of a wall-normal derivative of Eq. (14). Further, as the time-mean of fluctuating gradients also appears in the latter equation, simultaneous measurements are needed at two or more points in the wall-normal direction. The physical significance of the terms composing the Eq. (14) is briefly addressed in the next section.

4. Discussion

The expressions obtained in the previous section, which allow the experimental estimation of the amplitude of the fluctuating wall heat flux in a two-dimensional boundary layer, reveal the important role of the effusivity of the fluid in the unsteady heat transfer process as this quantity is a multiplicative factor affecting all terms in Eq. (14). As might be anticipated, these terms disclose both diffusive (conductive) and convective contributions to the RMS value of the fluctuating heat flux transferred at the wall.

The heat flux at $y = \delta y$ is essentially conductive if fluid velocities are negligible [12], and Eq. (14) simply becomes

$$\Phi'_{\delta y} \approx k\sqrt{\left(\frac{1}{2}\frac{\overline{\partial^2 (T'(t))^2}}{\partial y^2}\right)_{\delta y}} \tag{15}$$

In this case, the present energy balance leads to a result similar to that obtained in the analysis of the one-dimensional, transient heat conduction problem in a solid. As a consequence, these well-established results may be applied to the fluid side as well: the instantaneous temperature profile and the wall heat flux can be directly related by an algebraic expression involving the error function [3], also depending on the effusivity of the fluid [13].

On the other hand, if convection dominates over conduction, Eq. (14) takes the form

$$\Phi'_{\delta y} \approx e\sqrt{\overline{\left|T'(t)\left(\overline{U}\frac{\partial T'(t)}{\partial x}+V'(t)\frac{\partial T(t)}{\partial y}\right)\right|_{\delta y}}}, \tag{16}$$

where the modulus has been used to keep the correct sign of the quantity inside the square root. In this case, two different contributions to the RMS of the fluctuating wall heat flux may be identified. The first involves a correlation between temperature fluctuations, its derivative in the main flow direction and the mean flow velocity. The second is due to a correlation between temperature fluctuations, and the wall-normal temperature gradient and velocity fluctuations. The fluid effusivity appears as a multiplicative factor in both contributions, thus confirming its role in unsteady heat transfer processes by forced convection. It follows that these are primarily controlled by the product of fluid density, specific heat capacity and thermal conductivity. Further, we may conclude that augmented heat transfer rates will occur not only as a consequence of the simultaneous presence in the flow of fluctuations of temperature and wall-normal velocity. In fact, the coupling between a longitudinal gradient of temperature fluctuations and the local value of the mean flow velocity $\overline{U}$ may also be responsible by the occurrence of increased values of the wall flux. Hence, Eqs. (14)–(16) can be used to analyze how the fluid flow quantities affect the amplitude of the wall heat flux, e.g., for unsteady heat transfer optimization studies purposes in boundary layers. Ultimately, given the proper experimental implementation, estimates of the amplitude of the wall heat flux can also be obtained, carefully taking into account the range of validity of the underlying approximations.

5. Conclusions

A novel expression that, given the proper experimental implementation, may allow us to obtain estimates of the amplitude of the fluctuating heat flux at the wall in a two-dimensional, turbulent boundary layer has been presented. It involves the flow parameters and fluid variables, namely the temporal fluctuating velocity and temperature fields, as well as the fluid effusivity (entailing a product of fluid density, specific heat capacity and thermal conductivity). A number of requirements for the accurate measurement of the heat flux using this expression have been identified, namely: constant fluid properties; boundary layer flow; flow and temperature steady in the mean sense; measurements in two or more points in wall-normal direction. The global accuracy of the estimates is ultimately determined by the magnitude of the remainder term of the Taylor series, thus demanding measurements very close to the wall. The analysis of the proposed expression for the wall heat flux also revealed which flow characteristics are affecting the different modes of unsteady heat transfer that may occur. To summarize, we have proposed an alternative methodology to study the unsteady heat transfer at a wall taking into account the flow dynamics, instead of the classical approach of adopting the solid's perspective. The methodology has the potential and the benefit of providing more accurate estimates of the unsteady heat flux at the wall, as the problems resulting from a deficient contact to the wall substrate and damping of temperature oscillations due to the thermal capacitance of the solid wall are circumvented.

Appendix A

The unsteady heat flux at the wall $\Phi_w(t)$ can be extrapolated at the location $y = \delta y (\delta y > 0)$ retaining the first-order terms in a Taylor series development

$$\Phi_w(t) = -k\left(\frac{\partial T(t)}{\partial y}\right)_{y=0} \approx -k\left(\frac{\partial T(t)}{\partial y}\right)_{y=\delta y} + k\left(\frac{\partial^2 T(t)}{\partial y^2}\right)_{y=\delta y}\delta y$$
$$\approx \Phi_{\delta y}(t) - \frac{\partial \Phi_{\delta y}(t)}{\partial y}\delta y \tag{17}$$

A similar procedure may be applied to the fluctuating part of the instantaneous wall flux

$$\Phi'_w(t) \approx \Phi'_{\delta y}(t) - \frac{\partial \Phi'_{\delta y}(t)}{\partial y}\delta y \tag{18}$$

which, using the derivation rule, may be squared as follows:

$$(\Phi'_w(t))^2 \approx \left(\Phi'_{\delta y}(t)\right)^2 - \frac{\partial}{\partial y}\left(\left(\Phi'_{\delta y}(t)\right)^2\right)\delta y + \left(\frac{\partial \Phi'_{\delta y}(t)}{\partial y}\right)^2 \delta y^2 \tag{19}$$

The wall distance δy must be small enough to allow the following order of magnitude analysis:

$$\left(\frac{\partial \Phi'_{\delta y}(t)}{\partial y}\right)^2 \delta y^2 \ll \left(\Phi'_{\delta y}(t)\right)^2 - \frac{\partial}{\partial y}\left(\left(\Phi'_{\delta y}(t)\right)^2\right)\delta y \tag{20}$$

Hence, Eq. (19) may be simplified to obtain

$$(\Phi'_w(t))^2 \approx \left(\Phi'_{\delta y}(t)\right)^2 - \frac{\partial}{\partial y}\left(\left(\Phi'_{\delta y}(t)\right)^2\right)\delta y \tag{21}$$

Finally, taking the time-average of Eq. (21), yields

$$\overline{(\Phi'_w(t))^2} \approx \overline{\left(\Phi'_{\delta y}(t)\right)^2} - \frac{\partial}{\partial y}\left(\overline{\left(\Phi'_{\delta y}(t)\right)^2}\right)\delta y \tag{22}$$

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